# Investigation on factorization preconditioners
(created in 04/18/2017)

The investigated matrix is generated from abaqus software for SSI interaction model. The model includes two parts: 30X30X30m soil domain and a small 3-story structure, total dof is 34422.

After assembling the element matrices, Newmark effectivestiffness matrix is constructed to solve the linear system of equations.

Sparsity pattern of the effectivestiffness matrix
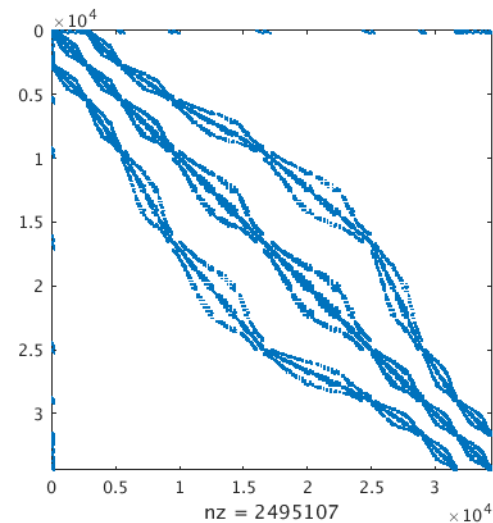
The condition number of effectivestiffness matrix(positive definite) is 1e13(which is very bad, usually around 1e6, 1e8) determines the convergence rate of the problem and also the number of accurate digits can be obtained(Timothy Sauer's numerical book).

Both incomplete LU and incomplete cholesky factorization are investigated inside the preconditioned conjuagate gradient method in Matlab.

ILU in matlab (recommended for BICG, GMRES):
5 options:
(1) type: no fill, crout, ilutp
(2) droptol:
(3) milu: off, row, column
(4) udiag: 0, 1
(5) thresh

No fill	1.8G, can not converge at 500 iterations
No fill, milu(row)	ill conditioned preconditioner
No fill, milu(col)	Most run to 1000 iterations but stagnated
Crout, droptol 1e-3	4G, pcg converge at iteration around 100, some stagnated

Crout, droptol 1e-4	10G, pcg converge at iteration around 50, some stagnated
Crout, droptol 1e-5	22G, pcg converge at iteration around 50, some stagnated
Crout, droptol 1e-3, milu(row)	5G, pcg can not converge in 5000 iterations
Crout, droptol 1e-3, milu(column)	5G, pcg can not converge in 5000 iterations

ICHOL in matlab (recommended for PCG, MINRES)
5 options:
(1) type: no fill, crout, ilutp
(2) droptol:
(3) milu: off, row, column
(4) udiag: 0, 1
(5) thresh

No fill	Non-positive pivot
No fill, michol on	Non-positvie pivot
No fill, diagcomp, alpha	Can not converge at 5000th iterations
ICT, droptol 1e-2	Non-positive pivot
ICT, droptol 1e-3	1.8G, most converges at 50 iterations, some stagnated at 50th
ICT, droptol 1e-4	8G, most converges at 50 iterations, some stagnated at 50th
ICT, droptol 1e-5	14G, most converges at 50 iterations, some stagnated at 50th
ICT, droptol 1e-6	28G, most converges at 50 iterations, some stagnated at 50th
ICT, droptol 1e-3, diagcomp, alpha	1.8G, cannot converge in 5000 iterations
ICT, droptol 1e-6, michol on	Nonpositive pivot

Conclusion:

By trying ILU and ICHOL preconditioners in matlab with all possible options, it is found that incomplete cholesky factorization with dropping tolerance is the best possible preconditioner.

ILU is not suitable for positive difinite symmetric system while ICHOL is extremly suitable for finite element matrix, it is also written in KJ Bathe's book.